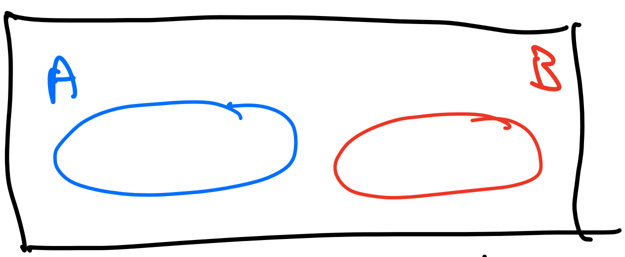
Intro: learn:

basics of boolean logic
about "programmable logic"
what is a Field Programmable Gate Array (FPGA)
program FPGA
use Raspberry Pi computer (assume you know Python)
connect FPGA to RPi
voila ⇒ cheap and powerful DAQ system

Venn diagrams: good way to visualize logic

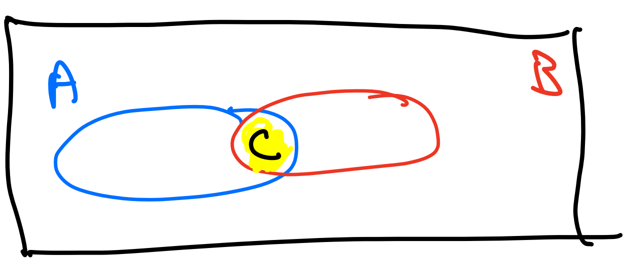
John Venn 1834-1923
wrote "The Logic of Chance" 1866, modern probability



Venn diagrams (1880)

"the Universe" U

Any member in universe can be in A, B, both, neither



Now let A & B intersect

ex: A = set of republicans in the universe

B = set of women in universe

then C = set of women who are also republicans

so $C = A$ "and" B can be written w/ symbols:

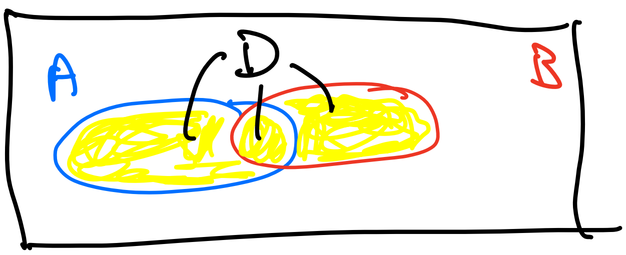
$C = A \cap B$ $\quad\quad$ $\cap$ = "intersection"

$C = A \& B$ $\quad\quad$ $\&$ = "and"

$C = A \cdot B$ $\quad\quad$ $\cdot$ = "and"

shorthand: $C = AB$ ($\cdot$ is implicit)

Next let D = set of elements who are either women or republicans

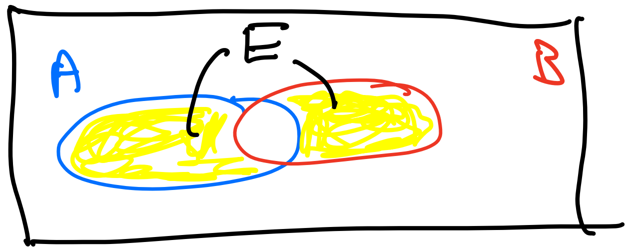


$D = A \cup B$ $\quad\quad$ $\cup$ = "union"

$= A \mid B$ $\quad\quad$ $\mid$ = "or"

$= A + B$ $\quad\quad$ $+$ = "or"

Next let E = elements who are either rep or women but <u>not</u> both



$E =$ "exclusive or" of A, B

$= A \oplus B \qquad \oplus =$ "exclusive or"

so we have $\quad C = A \cdot B$

$D = A + B$ "or"

$E = A \oplus B$

then we can write $\quad D = C + E$

or $\quad A + B = A \cdot B + A \oplus B$

Bayesian statistics

probability that an element is

$P(\text{GOP}) \equiv P(A) = \dfrac{A}{U}$ ratio of areas interpreted as a probability

$P(♀) \equiv P(B) = \dfrac{B}{U}$

$P(\text{female and GOP}) = \dfrac{A \cdot B}{U}$

what is "conditional probability" that

any ♀ is also a GOP?

denoted $P(B|A)$ "prob of GOP given ♀"

$$P(B|A) = \frac{A \cdot B}{A} \Leftarrow \frac{\text{GOP} \,\&\, ♀}{♀}$$

also, prob that any GOP is also a female?

$$P(A|B) = \frac{A \cdot B}{B}$$

then write $A \cdot B = B\,P(A|B) = A\,P(B|A)$

divide by U: $\underbrace{\frac{B}{U}}_{P(B)} \cdot P(A|B) = \underbrace{\frac{A}{U}}_{P(A)} P(B|A)$

this gives us Bayes thm (understanding probabilities when given prior information)

$$\frac{P(A|B)}{P(A)} = \frac{P(B|A)}{P(B)}$$

Rev. Thomas Bayes, 1701-1761

Boolean algebra George Boole 1815-1864

wrote "Laws of Thought" 1854

lays out the algebra of reasoning!

as above: $A+B = (A \cdot B) + (A \oplus B)$

algebra's have properties: For Boolean:

Commutative

$A+B = B+A$ & $A \cdot B = B \cdot A$

Distributive

$A + (B+C) = (A+B)+C$ & $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

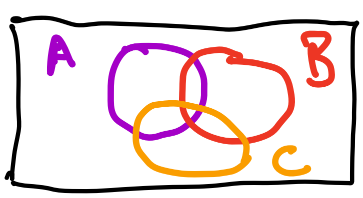
$= A+B+C$ $= A \cdot B \cdot C$

Associative

$A + (B \cdot C) = (A+B) \cdot (A+C)$

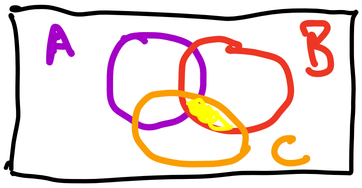
$A \cdot (B+C) = (A \cdot B) + (A \cdot C)$

easy to prove using Venn diagrams

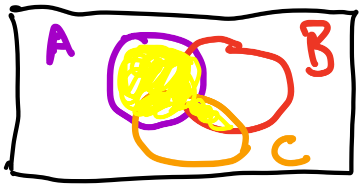


$A + (B \cdot C) = (A+B) \cdot (A+C)$

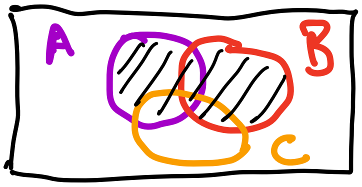
first do $A + (B \cdot C)$



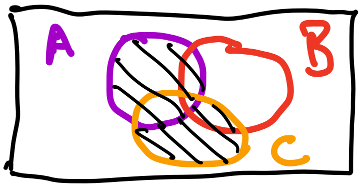
$B \cdot C$



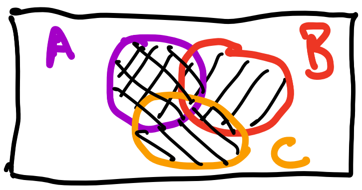
$A + (B \cdot C)$



$A + B$



$A + C$



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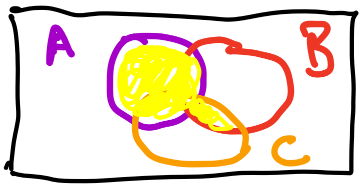


✓

Properties can be useful simplifying. eg:

$$D = (A \cdot B) + (B + C) \cdot (B \cdot C) = (A \cdot B) + B \cdot (B \cdot C) + C(B \cdot C)$$

then $B \cdot (B \cdot C) = B \cdot B \cdot C = B \cdot C$

$C \cdot (B \cdot C) = C \cdot B \cdot C = B \cdot C$

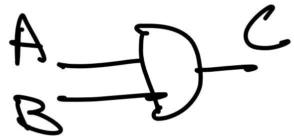
so $D = A \cdot B + BC + B \cdot C = A \cdot B + B \cdot C = B \cdot A + B \cdot C$

$= B \cdot (A + C)$

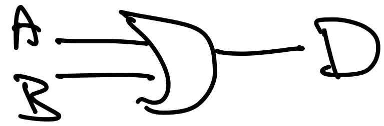
Digital representation

GATES:

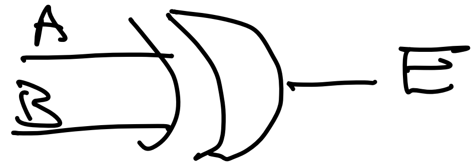
AND Gate $C = A \cdot B$



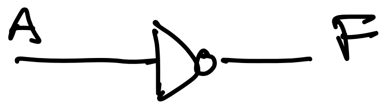
OR " $D = A + B$



XOR " $E = A \oplus B$



NOT " $F = \bar{A}$



TRUTH TABLE:

A B	$A \cdot B$	$A + B$	$A \oplus B$	$\bar{A}$	$\bar{B}$
0 0	0	0	0	1	1
0 1	0	1	1	1	0
1 0	0	1	1	0	1
1 1	1	1	0	0	0

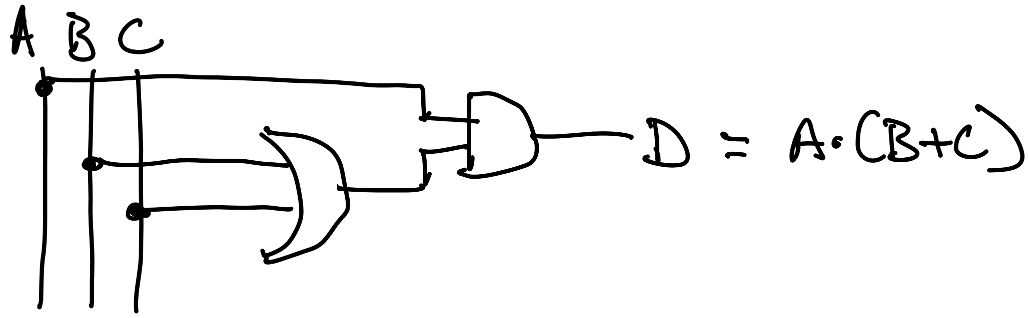
can prove distributive property using truth tables

Prove $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

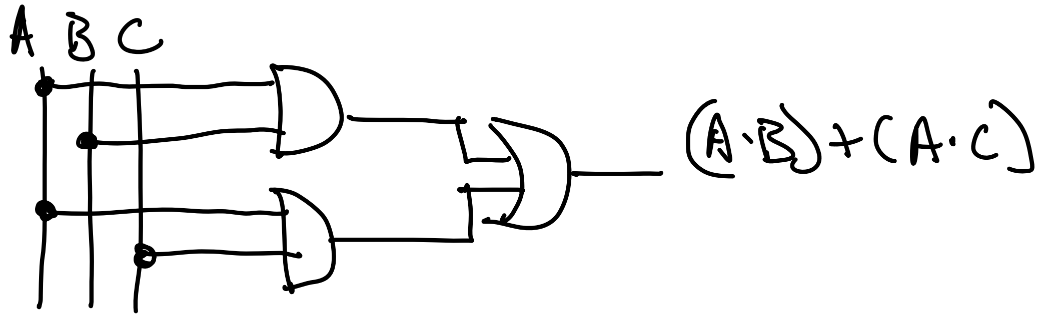
A B C	B+C	A(B+C)	A·B	A·C	(A·B)+(A·C)
0 0 0	0	0	0	0	0
0 0 1	1	0	0	0	0
0 1 0	1	0	0	0	0
0 1 1	1	0	0	0	0
1 0 0	0	0	0	0	0
1 0 1	1	1	0	1	1
1 1 0	1	1	1	0	1
1 1 1	1	1	1	1	1

Network of gates

ex:



ex:



they are the same (have same output)

Boolean properties of gates

involution: $\overline{\overline{A}} = A$ double inversion

idempotency: $A + A = A$ & $A \cdot A = A$

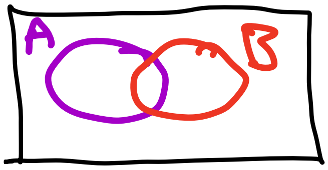
$A + \overline{A} = 1$ $\quad A + 0 = A$

$A \cdot \overline{A} = 0$ $\quad A \cdot 1 = A$

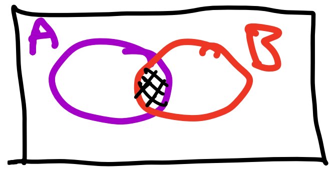
absorption: $A + (A \cdot B) = (A + A) \cdot (A + B) = A \cdot (A + B)$

$= (A \cdot A) + (A \cdot B) = A + A \cdot B = A$!

Venn makes it easy:



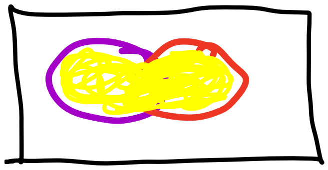
then $A \cdot B =$



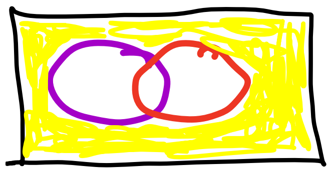
if you then take A+ you get A

De Morgan's thm (August De Morgan, 1806-1871)

$A+B =$



$\overline{A+B} =$



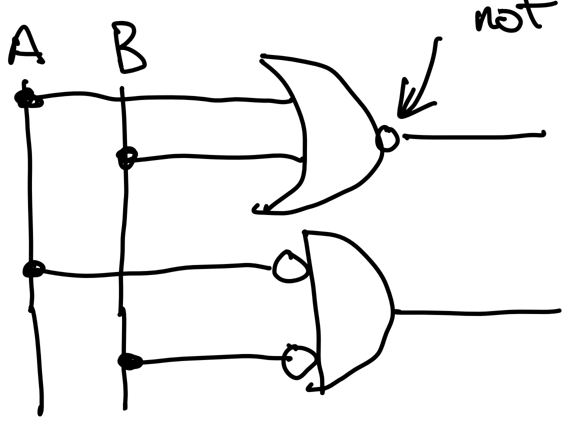
not in (A or B)

not in (A or B) means not A <u>and</u> not B $= \bar{A} \cdot \bar{B}$

so $\overline{A+B} = \bar{A} \cdot \bar{B}$ DeM's theorem

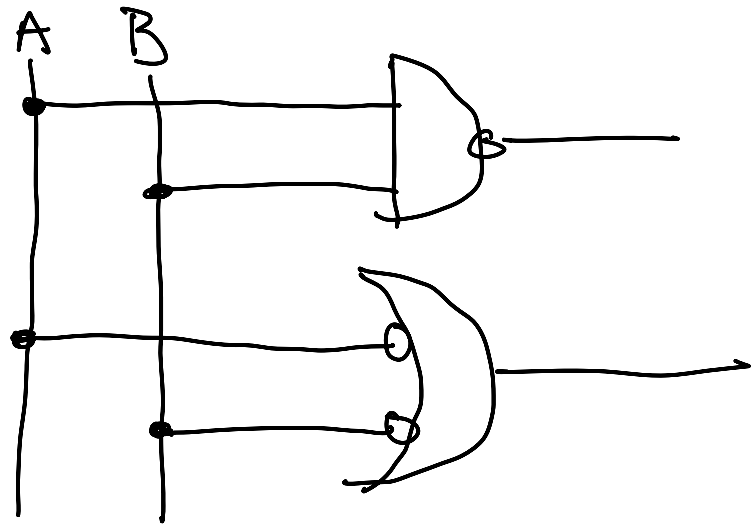
we use it for negating

not ≡ invert



equivalent!

DeM's thm: take gate, negate all inputs & outputs, and swap AND ⇔ OR so $\overline{A \cdot B} = \overline{A} + \overline{B}$



ex for using DeM's thm:

$$A \oplus B = \underbrace{(A+B)}_{A \text{ or } B} \cdot \underbrace{(\overline{A \cdot B})}_{\text{not both}}$$

$$= (A+B) \cdot (\overline{A}+\overline{B}) \quad \text{DeM}$$

$$= A \cdot (\overline{A}+\overline{B}) + B \cdot (\overline{A}+\overline{B})$$

$$= A \cdot \overline{A} + A \cdot \overline{B} + B \cdot \overline{A} + B \cdot \overline{B}$$

$$= A \cdot \overline{B} + B \cdot \overline{A}$$

let's drop the $\cdot$ and write $A \oplus B = A\overline{B} + B\overline{A}$
($\cdot$ implicit)